"Research problems on chaotic advection in three dimensions and at higher Reynolds number".

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Fourth annual technical report for period 5/01/93 - 6/30/94 and final technical report

The original grant was awarded to the PI at Columbia University for the two year period 5/01/90 - 4/30/92. Following the PI's move in 1992 to the University of Arizona to become Head of the Applied Mathematics Program, two no-fund extensions were granted for the periods 5/01/92 - 4/30/93 and 5/01/93 - 6/30/94 to enable the residual funds to be used for a number of grant related projects. This report describes the fourth and final year of activities and sums up the entire effort.

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6. AUTHOR(S)
M. Tabor
I. Klapper

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Columbus Universityin the City of New York
Box 20, Low Memorial Library
New York, NY 10027

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NA
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Over the past ten years the study of chaotic advection, namely the chaotic motion of particles in deterministic dynamical systems derived from velocity fields associated with simple fluid flows, has come to the fore as a model and means of obtaining efficient mixing. The main question addressed in the research supported by this grant has been to determine how useful the ideas and tools of dynamical system theory - successful in analyzing chaotic advection models - are for studying mixing in high Reynolds number, i.e. turbulent, flows. The link between dynamical systems ideas and turbulence is provided by the Lagrangian picture of the fluid, namely the fluid particle description. Accordingly our work has centered on developing Lagrangian descriptions of the stretching and alignment - the processes by which mixing is achieved - for passive and non-passive scalars and vectors. This has led to some new insights into the mechanisms of fine scale vorticity dynamics and identified the subtle and critical role of pressure fluctuations. In the case of the (non-passive vector) dynamics of magnetic field lines, a Lagrangian formulation of 3-dimensional magneto-hydrodynamics equations has identified the possibility of a finite time singularity around magnetic null points.

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Work performed during last year of project.

The principal theme of research has been the study of chaotic advection in three dimensions at high Reynolds numbers. During the first two years this took the form of studies of the Lagrangian evolution of vorticity and strain and their role in stretching, aligning and mixing of both passive scalars - and hence chaotic mixing - and vorticity itself. A detailed analysis of vorticity stretching and alignment led to some new insights into the generation of small scale vorticity in turbulence and the subtle role of pressure fluctuations. During the past year two main directions have been pursued: (i) the nature of pressure fluctuations in turbulent flows and (ii) the stretching and alignment of non passive quantities.

<u>Pressure fluctuations and structures in turbulence</u>. Work by graduate student Emanuel Leveque (supervised by Z-S She) has, by direct visualization and conditional sampling analysis, studied the correlation between high magnitude structures for coarse grained dissipation and the vorticity and the low amplitude structures of the pressure fluctuations. The studies show that this correlation becomes more noticeable as the Reynolds number increases and that the structures tend to filaments. (This is different from the past claims that have been made for more sheet like structures.)
Additional work has also involved an exploration of the eigenstates of the pressure Hessian which determines the propagation of vorticity disturbances.

<u>Stretching and Alignment of non-passive quantities.</u> During the past year we have extended our studies of stretching and mixing of scalar quantities to the cases of passive vectors such as scalar gradients and, in addition to vorticity, other *non-passive vectors* such as magnetic field lines in fully coupled magneto-hydrodynamics (i.e in which the magnetic field and velocity field motions are coupled) . Many of these ideas are reviewed in an invited review article "Stretching and alignment in chaotic and turbulent flows" written with post-doctoral colleague Isaac Klapper. A particular emphasis was placed on the stretching of magnetic field lines. In work with Klapper and graduate student Anita Rado, a new Lagrangian formulation of the 3 dimensional MHD equations was developed. By making certain reasonable assumptions about the pressure field (namely, locally isotropic) a closed system of odes (of traces of the relevant tensors) is obtained which can be subjected to detailed numerical and analytical studies. Our studies reveal the formation of a finite time singularity at null points and the use of

various analytic continuation techniques have identified the detailed structure of the singularity itself.

Papers published during final year

M. Tabor and I. Klapper, "Stretching and alignment in chaotic and turbulent flows", *Chaos, Solitons and Fractals*, 1031 - 1055, **4** (1994), special volume edited by H. Aref.

Manuscripts in preparation

I. Klapper, A. Rado and M. Tabor, "A Lagrangian study of singularity formation at magnetic null points in ideal 3 dimensional magneto-hydrodynamics"

E. Leveque and Z.-S. She "Structure of dissipation, vorticity and pressure fluctuations in isotropic turbulence"

Overall project summary

Over the past ten years the study of chaotic advection, namely the chaotic motion of particles in deterministic dynamical systems derived from velocity fields associated with simple fluid flows, has come to the fore as a model and means of obtaining efficient mixing. The main question addressed in the research supported by this grant has been to determine how useful the ideas and tools of dynamical system theory - successful in analyzing chaotic advection models - are for studying mixing in high Reynolds number, i.e. turbulent, flows. The link between dynamical systems ideas and turbulence is provided by the Lagrangian picture of the fluid, namely the fluid particle description. Accordingly our work has centered on developing Lagrangian descriptions of the stretching and alignment - the processes by which mixing is achieved - for passive and non-passive scalars and vectors. This has led to some new insights into the mechanisms of fine scale vorticity dynamics and identified the subtle and critical role of pressure fluctuations. In the case of the (non-passive vector) dynamics of magnetic field lines, a Lagrangian formulation of 3-dimensional magneto-hydrodynamics equations has identified the possibility of a finite time singularity around magnetic null points.

Published papers

E. Dresselhaus and M. Tabor, "The Kinematics of Stretching and Alignment of Material Elements in General Flow Fields", J. Fluid. Mech. **236**, 415 - 444 (1991).

M. Tabor, "Stretching and alignment in general flow fields: classical trajectories from Reynolds number zero to infinity" in "Topological aspects of the dynamics of fluids and plasmas", NATO ASI series E, **218** (editors: H. K. Moffatt, G. M. Zaslavsky, P. Comte and M. Tabor; Kluwer Academic Publishers, 1992).

M. Tabor and I. Klapper, "Stretching and alignment in chaotic and turbulent flows", *Chaos, Solitons and Fractals* **4**, 1031 - 1055 (1994), special volume edited by H. Aref.

Manuscripts in preparation

I. Klapper, A. Rado and M. Tabor, "A Lagrangian study of singularity formation at magnetic null points in ideal 3 dimensional magneto-hydrodynamics"

E. Leveque and Z.-S. She "Structure of dissipation, vorticity and pressure fluctuations in isotropic turbulence"

Research report

E. Dresselhaus and M. Tabor, "The genesis of small scale vorticity in turbulence" (1992).

Dissertations

E. Dresselhaus, "Stretching and Alignment in Turbulence", Department of Applied Physics, Columbia University, New York (1991).

Invited presentations of research supported by grant

2/22/91 University of Colorado, Boulder, Applied Mathematics Colloquium, "Stretching and Alignment in Turbulence"

2/25/91 University of Texas, Austin, Nonlinear Dynamics Colloquium, "The Kinematics of Stretching and Alignment"

3/4/91 Princeton University, Applied Mathematics Colloquium, "Stretching and Alignment in Turbulence"

3/6/91 City College New York, Physics Colloquium, "Stretching and Alignment in Turbulence"

3/21/91 APS Meeting Cincinnati, session on Fluid Mechanics (H. Swinney, chair), "Stretching and Alignment in Turbulence"

5/7/91 CEN, Saclay, Physics seminar, "Stretching and Alignment in Turbulence"

6/25/91 (Pt I) & 6/28/91 (Pt II) Observatoire de Nice, seminar on "The Kinematics of Stretching and Alignment in Turbulence"

10/28/91 U. C. Santa Cruz, Physics Colloquium, "Stretching and Alignment in Turbulence"

5/13/92 Utah State University, Mathematics Colloquium, "Stretching and Alignment in Turbulence"

7/16/92 Australian National University, Canberra, Physics Colloquium, "Stretching and
Alignment in Turbulence"

8/3/92 Univ. of New South Wales, Applied Mathematics Colloquium, "Stretching and Alignment in Turbulence"